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VOL. 23, SEC. A. MARCH, 1945 NUMBER 2

A THEORY OF THE CREATION OF ELECTRIC CHARGE[1]

BY W. H. WATSON[2]

Abstract

The theory is based on equations previously proposed (2) associating with ordinary electromagnetic fields a field which may change the rest-mass of the particles on which it acts. The sources of the new field component $\left(N = -\operatorname{div} \boldsymbol{a} - \frac{1}{c^2}\frac{\partial \varphi}{\partial t}\right)$ are places where charge is being created or destroyed. A new interpretation is given to the length e^2/mc^2, and it is proposed that the e.m. potentials $(\varphi, \boldsymbol{a})$ be given absolute values in the theory of fields containing electrons. Thus the mass of an electron is determined by the length of the potential four-vector at the place where it is created, and

$$m c = e\sqrt{\frac{\varphi^2}{c^2} - \boldsymbol{a}^2}.$$

The introduction of the N-component into quantum electrodynamics is discussed.

Equations of the System

That electric charge can be created or destroyed is a conception foreign to the Maxwell–Lorentz theory of electricity, and certainly our experience of electrical systems on the molar scale shows no need of the idea. On the other hand, one can point to the phenomena of pair-production and of β-emission and there one seems forced to admit discontinuous processes of creation and destruction.

The ideas to be presented here were derived in the first place by associating with ordinary electromagnetic fields of classical type, a new type of field which may change the rest-mass of particles on which it acts (2). An electromagnetic field modified to include this effect is represented by a tensor of which the diagonal components do not vanish, but instead are equal to a scalar N which satisfies the equation

$$\nabla^2 N - \frac{1}{c^2}\frac{\partial^2 N}{\partial t^2} \equiv \Box N = \mu_0\left(\frac{\partial \rho}{\partial t} + \operatorname{div} \boldsymbol{j}\right), \tag{1}$$

where ρ and $\boldsymbol{j}$ are respectively the charge and current densities in M.K.S. units. The latter have been redefined so as to take into account the variation of N in space and time. The fundamental equations of this new system are

$$\operatorname{curl} \boldsymbol{E} = -\dot{\boldsymbol{B}}, \qquad \operatorname{div} \boldsymbol{B} = 0, \tag{2}$$

[1] *Manuscript received September 29, 1944.*
Contribution from the Department of Mathematics, University of Saskatchewan, Saskatoon, Sask.

[2] *Professor.*

so that there exist potentials $\boldsymbol{a}$ and φ, such that

$$\boldsymbol{E} = -\dot{\boldsymbol{a}} - \text{grad } \varphi, \qquad \boldsymbol{B} = \text{curl } \boldsymbol{a}. \tag{3}$$

The densities $\boldsymbol{j}$ and ρ are defined by

$$\boldsymbol{j} = \text{curl } \boldsymbol{H} - \dot{\boldsymbol{D}} + \frac{1}{\mu_0} \text{grad } N, \qquad \rho = \text{div } \boldsymbol{D} - \kappa_0 \dot{N}, \tag{4}$$

and we have the usual relations $\boldsymbol{B} = \mu_0 \boldsymbol{H}$, $\boldsymbol{D} = \kappa_0 \boldsymbol{E}$. It is seen that Equation (1) follows from the pair (4). The equations satisfied by the potentials may still be written in their usual form

$$\Box \boldsymbol{a} = -\mu_0 \boldsymbol{j}, \qquad \Box \varphi = -\frac{\rho}{\kappa_0}, \tag{5}$$

provided that

$$\text{div } \boldsymbol{a} + \frac{1}{c^2} \dot{\varphi} = -N. \tag{6}$$

The energy-density in the field is

$$\tfrac{1}{2} (\mu_0 H^2 + \kappa_0 E^2 + \frac{1}{\mu_0} N^2),$$

and the energy-flux $\boldsymbol{E} \times \boldsymbol{H} - \frac{1}{\mu_0} \boldsymbol{E} N$.

The following are the equations of motion of a particle of charge q, rest-mass m, whose position is denoted by the vector $\boldsymbol{r}$.

$$\left.\begin{aligned} \frac{d\boldsymbol{P}}{ds} &= q\left\{ N \frac{d\boldsymbol{r}}{ds} + \boldsymbol{E} \frac{dt}{ds} + \frac{d\boldsymbol{r}}{ds} \times \boldsymbol{B} \right\} \\ \frac{dP_4}{ds} &= q\left\{ c^2 N \frac{dt}{ds} + \boldsymbol{E} \cdot \frac{d\boldsymbol{r}}{ds} \right\} \end{aligned}\right\} \tag{7}$$

s denotes the proper time at the particle, and

$$\boldsymbol{P} = m \frac{d\boldsymbol{r}}{ds}, \qquad \frac{1}{c^2} P_4^2 - \boldsymbol{P}^2 = m^2 c^2. \tag{8}$$

From Equation (7) we deduce

$$qN = \frac{dm}{ds} \tag{9}$$

and

$$m \frac{d^2 \boldsymbol{r}}{ds^2} = q\left(\boldsymbol{E} \frac{dt}{ds} + \frac{d\boldsymbol{r}}{ds} \times \boldsymbol{B} \right). \tag{10}$$

If q/m is constant, it is possible to construct from Equation (10) a Hamiltonian system which is quite analogous to that for a particle of fixed charge and mass acted on by the usual type of electromagnetic field; velocity now plays the part of momentum. However, this system is too restricted to represent electron–mesotron transformations.

Energy and Momentum Relations

From Equation (1) it follows that if a charge q' at rest at the point Q, changes with time, it will produce at P, distant r from Q, a field with N-component

$$N = -\frac{\mu_0}{4\pi r}[\dot{q}'], \tag{11}$$

where the [ ] denote that $\dot{q}'$ is to be calculated at time r/c antecedent to that at which the value of N is considered. Consequently if a changing charge q is placed at P, and if we denote by m and m' the respective masses of the particles at P and Q, we have

$$\frac{dm}{ds} = -\frac{\mu_0 q}{4\pi r}[\dot{q}'] \qquad \frac{dm'}{ds} = -\frac{\mu_0 q'}{4\pi r}[\dot{q}] \tag{12}$$

Now, in accordance with Heisenberg's uncertainty principle, there is a limit to the precision with which the variation of m can be followed in time, and since we wish to speak of the total mass of the pair of particles, so that it is a matter of indifference which is measured first, it is out of place in a crude theory to introduce the refinement of retarded action; we therefore omit it and on integration obtain for the change in the total rest-mass

$$\Delta(m + m')c^2 = -\Delta\frac{qq'}{4\pi r \kappa_0} \tag{13}$$

By this semiclassical procedure, we reach the result that a pair of *unlike* charges brought into existence at two separate points will have a total mass equal to minus their mutual potential energy (referred to infinite separation) divided by c^2. *Aliter*, Two charges without mass at infinity, if allowed to come close together and brought to relative rest, must acquire mass equal to the loss of potential energy divided by c^2. The mass might be said to be due to the non-electromagnetic forces required to bring the particles to rest.

If the two particles are equally massive and are created with equal and opposite charges q, the mass of each is

$$m = q^2/8\pi\kappa_0 c^2 a \quad \text{(or in E.S.U.} \quad e^2/2ac^2), \tag{14}$$

where a is the distance between the two points at which the particles are created. Thus a new interpretation is given to the length e^2/mc^2: it need no longer enter physical theory as the electron radius, characteristic of an electron by itself: it denotes a relation in which the electron at creation stands with respect to its environment, just as the proper mass must. Another way in which the relation can be expressed is to write

$$mc^2 = e\varphi_0 \tag{15}$$

and to suppose that the electromagnetic scalar potential (and with it, of course, the vector potential) have to be given absolute values in the theory of electromagnetic fields containing electrons. This idea is to be contrasted with the

classical theory in which the electric and magnetic field vectors only are specified, and the potentials, being defined by differential equations, are indeterminate.

It is proposed, therefore, that when an electric charge is created with velocity v and momentum $m\beta v$, and energy W we should write

$$mc = e\sqrt{\frac{\varphi_0^2}{c^2} - a_0^2}, \qquad p = m\beta v = ea_0, \qquad W = mc^2\beta = e\varphi_0. \quad (16)$$

This mode of expression emphasizes the relativity of momentum and energy, not only in their dependence on the Galilean frame of reference chosen for their measurement, but also on the physical environment represented in the theory by the electromagnetic field, in which the potentials are now determinate. From this point of view, it is the magnitude of the potential 4-vector at the place where the charge is created that determines the mass of the particle, while the components of the potential 4-vector fix the components of the energy–momentum 4-vector. Correspondingly, the annihilation of charge or mass would be expected when the moving charge reaches a place at which its total momentum is equal to its potential momentum.

As an application of these ideas, let us consider the neutron→proton transformation in a nucleus resulting in the emission of a β-particle. Think first of a β-particle emitted with very small kinetic energy: a neutron changes to a proton: if only one neutron were involved, and no other particles enter energetically in the transformation process, from Equation (13) we require

$$m_P - m_N + m_e = e^2/ac^2.$$

Now the left-hand side is known to be negative; hence there is no value of a at which the transformation can occur, and the same holds for the inverse transformation. A certain mass must be supplied and this can come only from the nucleus of which the proton or neutron forms a part. On the other hand, it would be possible for the transformations to take place outside a nucleus, if a more massive electron were emitted or absorbed. Other things being the same, one expects a more energetic particle to be emitted, the closer the point of its creation is to the nucleus.

Quantum Electrodynamics

In order to indicate how the N-component of the field may be introduced into wave-mechanics, consider Gordon's relativistic equation (without spin) which is written

$$\left[\left(\frac{H - e\varphi}{c}\right)^2 - (p - ea)^2\right]\psi = m^2c^2\psi, \quad (17)$$

H and p denoting the usual differential operators. It is seen at once that the quantity $e\left(\frac{1}{c^2}\frac{\partial\varphi}{\partial t} + \text{div } a\right)$ emerges from the operations. This quantity has

usually been taken to vanish, but according to Equation (6) above, we should write it as $-eN$. Equation (17) becomes

$$\left[-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{4\pi ie}{hc^2}\varphi\frac{\partial}{\partial t}+\nabla^2+\frac{4\pi ie}{h}\boldsymbol{a}\cdot\nabla+\frac{4\pi^2e^2}{h^2}\left(\frac{\varphi^2}{c^2}-\boldsymbol{a}^2\right)\right]\psi = \left(\frac{4\pi^2m^2c^2}{h^2}-\frac{2\pi ieN}{h}\right)\psi .$$

If the particle is moving slowly, we can concentrate on the time dependence of ψ. To a first approximation

$$\frac{1}{\psi}\frac{\partial\psi}{\partial t} = 2\pi i\nu + g ,$$

where

$$\nu^2 = \frac{m^2c^2}{h^2}+\frac{g^2}{4\pi^2}$$

and

$$g = -\frac{Ne}{2h\nu} = -\frac{Ne}{2mc^2},$$

i.e.,

$$\psi \backsim e^{2\pi i\nu t-\frac{Ne}{2mc^2}t} .$$

Thus N determines the rate of growth or decay of the wave amplitude, and hence, by second quantisation changes the number of particles.

In introducing spin we may couple* Equation (6) above (defining the divergence of the potential four-vector) with the linear equation of Dirac's theory. Development would then proceed on a somewhat different basis from that of Dirac, Fock, and Podolsky (1) in which the divergence of the potential is made to vanish and, in addition to the half of Maxwell's equations connecting current and charge with the field, this is a condition on possible wave-functions. Actually the quantum electrodynamics of these writers contains points of formal similarity with the equation system (2)–(4) at the beginning of the present paper, in that in place of our (4), there appear (with different units) the conditions

$$\left(\text{curl } \boldsymbol{H}-\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}\right)\psi = \text{grad}\sum_1^n\frac{\epsilon_s}{4\pi}\Delta(X-X_s)\psi$$

$$(\text{div } \boldsymbol{E})\psi = -\frac{1}{c}\frac{\partial}{\partial t}\sum_1^n\frac{\epsilon_s}{4\pi}\Delta(X-X_s)\psi$$

on the possible wave-functions ψ. The four-dimensional vectors $X = (x, y, z, t)$, $X_s = (x_s, y_s, z_s, t_s)$ appear in the invariant delta function

$$\Delta(X) = \frac{1}{|r|}\left[\delta(|r|+ct)-\delta(|r|-ct)\right],$$

* *I am indebted to the referee for this suggestion.*

the suffix s refers to the particle of charge ϵ_s, t_s is the individual time of the sth particle and t is the field time. Now $\triangle X$ has the limiting form as $\sigma \to 0$ of the expression for N appearing in Equation (21) of the paper which follows (3), and in which $c\sigma$ does not vanish but is of the order of the electron radius. Dirac, Fock, and Podolsky proceed by considering what happens when there is a common time $T = t = t_s$ ($s = 1,2, \ldots . n$). Of course the delta function vanishes. But according to the point of view of the present paper, it is just this quantity which must not vanish if there is to be creation of electric charge.

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DISCONTINUOUS MOTION OF AN ELECTRIC PARTICLE[1]

By W. H. Watson[2]

Abstract

At the outset this paper deals with a world-track which consists of a discrete series of events at which the particle may be found. The electric potential may change very rapidly in the vicinity of the null-cone with its vertex at the point–instant singularity, without requiring the usual electric current to sustain the wave of potential, provided that the electromagnetic field tensor is revised in accordance with equations previously proposed (5) to represent the non-optical radiation of energy and momentum. Such forms of potential represent the switching on and off of charge. Inertial mass, energy, and momentum are associated with the wave. Different possibilities for discontinuous motion of an electric particle occur, depending on the modes of its successive creations and annihilations.

The model discussed exhibits the essential separation of space–time into two regions, in one of which the field is static and in the other it vanishes, and leads to new insight into the structure of electromagnetic field singularities as boundaries of the field which we may think of as propagated like electromagnetic waves in a wave-guide.

Connection of these ideas with quantum theory is briefly discussed.

Discontinuous Motion

By the assertion that the motion of a particle is continuous, one means at least that the co-ordinates specifying its position are continuous functions of the time and that the particle may be found at any instant whatever during the period of its existence, or, in the language of relativity, the world-line of the particle is a one-dimensional continuum. Out of the many interpretations which may be given to the expression 'discontinuous motion' we shall be concerned here at the outset with a world-track which consists of a discrete series of regularly spaced events at which the particle may be found; i.e., any other point–instant apart from the series in question is not a possible place in space–time for the particle. The process we have in mind is well represented by the appearance of a moving light produced by the successive switching on and off of the lamps in a stationary row of lamps. If the particle has charge and mass, its discontinuous motion involves point–instant singularities in the electromagnetic and gravitational fields in contradistinction to the line singularities characteristic of Kirchhoff's elementary solutions of the wave-equation (4, p. 67). In the electromagnetic case, such a series of singularities requires that a single one corresponds to the creation of electric charge at one point–instant and its annihilation at an infinitesimally later instant with the same space co-ordinates.

[1] *Manuscript received September 29, 1944.*
Contribution from the Department of Mathematics, University of Saskatchewan, Saskatoon, Sask.

[2] *Professor.*

Series of Singularities and Waves

We propose first to deal with the potential at $P(t,\mathbf{r})$ due to a series of elementary solutions of the wave-equation of strength λ, namely

$$\frac{\lambda}{c^2(t-t_1)^2-|\mathbf{r}-\mathbf{r}_1|^2}, \tag{1}$$

where
$$t_1 = \ldots\ldots -2\tau, -\tau, 0, \tau, 2\tau \ldots\ldots$$
$$\mathbf{r}_1 = \ldots\ldots -2\mathbf{u}\tau, -\mathbf{u}\tau, 0, \mathbf{u}\tau, 2\mathbf{u}\tau \ldots\ldots$$

and the series extends indefinitely into the past and the future. The potential in question is

$$\varphi = \sum_{n=-\infty}^{\infty} \frac{\lambda}{c^2(t-n\tau)^2-|\mathbf{r}-n\mathbf{u}\tau|^2} \tag{2}$$

$$= \sum_{n=-\infty}^{\infty} \frac{\lambda}{(c^2t^2-r^2)-2n\tau(c^2t-\mathbf{u}.\mathbf{r})+n^2\tau^2(c^2-u^2)}. \tag{3}$$

We shall leave aside for the moment the exceptional circumstances under which the denominator of one of the terms of the series may vanish.

Let
$$\tau^2(c^2-u^2) = a, \qquad \frac{c^2t-\mathbf{u}.\mathbf{r}}{\tau(c^2-u^2)} = b$$

and
$$b^2 - \frac{c^2t^2-r^2}{\tau^2(c^2-u^2)} = f^2. \tag{4}$$

Then

$$\varphi = \sum_{n=-\infty}^{\infty} \frac{\lambda}{a}\cdot\frac{1}{(n-b)^2-f^2}$$

$$= \frac{\lambda}{a}\sum_{n=-\infty}^{\infty}\frac{1}{2f}\left[\frac{1}{n-b-f}-\frac{1}{n-b+f}\right]$$

$$= \frac{\lambda}{2af}\left\{\sum_{n=1}^{\infty}\left(\frac{2(b+f)}{n^2-(b+f)^2}-\frac{2(b-f)}{n^2-(b-f)^2}\right)+\frac{2f}{b^2-f^2}\right\}$$

$$= \frac{\lambda\pi}{2af}(\cot\pi(b-f)-\cot\pi(b+f)). \tag{5}$$

If the origin were excluded as a singularity by displacing t_1 through the interval $\frac{1}{2}\tau$ along the t-axis, we should then have the potential

$$\varphi = \frac{\lambda\pi}{2af}(\tan\pi(b+f)-\tan\pi(b-f)). \tag{6}$$

Now let us introduce the uniformly translated co-ordinate system so that

$$\tau' = \tau\left(1-\frac{u^2}{c^2}\right)^{\frac{1}{2}}, \qquad b = \frac{t'}{\tau'}, \qquad f = \frac{r'}{c\tau'} \tag{7}$$

while Equations (5) and (6) become respectively

$$\varphi' = \frac{\lambda\pi}{2c\tau'r'}\left[\cot\frac{\pi}{\tau'}\left(t'-\frac{r'}{c}\right)-\cot\frac{\pi}{\tau'}\left(t'+\frac{r'}{c}\right)\right] \tag{8}$$

and

$$\varphi' = \frac{\lambda\pi}{2c\tau'r'}\left[\tan\frac{\pi}{\tau'}\left(t'+\frac{r'}{c}\right)-\tan\frac{\pi}{\tau'}\left(t'-\frac{r'}{c}\right)\right]. \tag{9}$$

Equations (8) and (9) yield well known forms of solution of the wave-equation (Ref. (1)), and one might expect that the retarded part of this potential would lead in the limit $\tau' \to 0$ and $\frac{\lambda\pi}{2c\tau'} \to q'$ to the usual electrostatic potential of the point charge q'; but clearly the cotangent or tangent does not tend to a limit unless τ' is imaginary, i.e., unless $u > c$.

Suppose $\tau' = i\sigma$, where σ is real. In order to keep the singularities on the imaginary axis only of the complex t-plane, we have to choose Equation (9) which becomes, dropping primes,

$$\varphi = \frac{\lambda\pi}{2c\sigma r}\left[\tanh\frac{\pi}{\sigma}\left(t + \frac{r}{c}\right) - \tanh\frac{\pi}{\sigma}\left(t - \frac{r}{c}\right)\right]. \tag{10}$$

When $\sigma \to 0$ and $\frac{\lambda\pi}{c\sigma} \to q$

$$\varphi = q/r, \quad \text{for} \quad \frac{r}{c} > t > -\frac{r}{c}$$

$$\varphi = 0, \quad \text{for} \quad t > \frac{r}{c} \quad \text{and} \quad t < -\frac{r}{c}.$$

There is a discontinuity in the potential along the null-cone with its vertex at the origin. When σ is not zero, the potential changes very rapidly in the vicinity of the null-cone, and according to Maxwell–Lorentz theory could be sustained only by an electric current, which would result in the condensation of charge to the origin at $t = 0$ and its immediate dispersal to infinity with the velocity of light. On the other hand, if we adopt the equations proposed by the writer (5), the current is replaced by the gradient of the diagonal component (N) of the field tensor. This results from

$$\text{grad } N + \nabla \times \boldsymbol{H} - \frac{1}{c}\dot{\boldsymbol{E}} = 0, \qquad \text{div } \boldsymbol{E} = \frac{1}{c}\dot{N}. \tag{11}$$

In fact, if $\boldsymbol{H} = 0$,

$$E_r = -\frac{\partial\varphi}{\partial r}, \qquad N = -\frac{1}{c}\frac{\partial\varphi}{\partial t} \tag{12}$$

and as pointed out in the work referred to, there must be transfer of momentum per unit area $-\frac{1}{4\pi}NE$—with the velocity c into the origin when t is negative and outwards from it when t is positive. To an outside observer, distant r from the origin, it is as if the field due to an electric charge at the origin were created at $t = -r/c$ and destroyed at $t = +r/c$.

Before proceeding to examine potential forms derived from Equation (10) we may remark that Frenkel (2, p. 175) has shown how to derive the Liénard–Wiechert retarded potential due to a point charge from that of a distribution of elementary solutions (1) of the wave-equation. In his discussion, the series is not summed but is replaced at once by an integral with respect to t over a closed path in the complex t-plane, which is introduced into the argument for analytical purposes only, and, as Frenkel frankly stated, without any attempt to give physical meaning and justification to the analytical processes. The

closed path of integration is required if one imposes the usual condition on the potentials,

$$\frac{1}{c}\frac{\partial \varphi}{\partial t} + \operatorname{div} \boldsymbol{a} = 0. \tag{13}$$

Now this condition is equivalent to asserting that there shall be no creation or destruction of electricity in the field (3), and it cannot be satisfied if there is really discontinuous motion of an electric charge. However, this consideration did not seriously concern Frenkel in his calculation for he was interested essentially in continuous motion and did not raise the question of passing from the discrete to the continuous by a limiting process.

We now consider some forms of potential derived from Equation (10).

(*a*) The potential $\frac{q}{2r}\left[\tanh\frac{\pi}{\sigma}\left(t+\frac{r}{c}\right)+\tanh\frac{\pi}{\sigma}\left(t-\frac{r}{c}\right)\right]$ (14)

corresponds to the sudden change of charge at the origin from $-q$ to $+q$. At a point distant r from the origin, the field of the charge disappears from $t = -r/c$ to $t = +r/c$.

(*b*) The potential $\frac{q}{2r}\left[\tanh\frac{\pi}{\sigma}\left(t-\frac{r}{c}\right)+1\right]$ (15)

corresponds to the creation of charge q at the origin at $t = 0$, its field being established by retarded action.

(*c*) The potential $\frac{q}{2r}\left[\tanh\frac{\pi}{\sigma}\left(t-\frac{r}{c}\right)-1\right]$ (16)

corresponds to the annihilation of the charge $-q$ at the origin at $t = 0$, its field being destroyed by retarded action.

(*d*) The potential $\frac{q}{2r}\left[\tanh\frac{\pi}{\sigma}\left(t+\frac{r}{c}\right)+1\right]$ (17)

corresponds to the creation of the charge $+q$ at the origin at $t = 0$, the field being established from infinity inwards.

(*e*) The potential $\frac{q}{2r}\left[\tanh\frac{\pi}{\sigma}\left(t+\frac{r}{c}\right)-1\right]$ (18)

corresponds to the annihilation of $-q$ at the origin at $t = 0$, the field being destroyed from infinity inwards.

We shall speak of (*b*) as creation from within and (*d*) as creation from outside, and corresponding expressions will be applied to (*c*) and (*e*) respectively.

Thus for the discontinuous motion of an electric particle different possibilities occur depending on the mode of its successive creations and annihilations. If the latter are both of the same type, (*b*) and (*c*) or (*d*) and (*e*), there must be a net radiation or condensation of energy; whereas if the singularity is created from outside and annihilated from within at times $t = 0, T, 2T, \ldots$ the energy may be reflected at $r = \frac{1}{2}cT$ and outside this sphere the field

remains undisturbed at the electrostatic value. Within the sphere, a wave travels from the centre to the surface and back again and repeats this process indefinitely during the lifetime of the particle. The time interval between successive reflections of the wave is not determined in our model and we have not considered how precisely the position and motion of the singularity may be fixed. Both of these questions are in the province of quantum theory. Before considering them, we shall examine further the field defined by Equation (10) with $\frac{\lambda\pi}{\sigma c} = q$, i.e.,

$$\varphi = \frac{q}{2r}\left[\tanh\frac{\pi}{\sigma}\left(t+\frac{r}{c}\right) - \tanh\frac{\pi}{\sigma}\left(t-\frac{r}{c}\right)\right]. \tag{19}$$

We have

$$E_r = -\frac{\partial\varphi}{\partial r} = \frac{q}{2r^2}\left[\tanh\frac{\pi}{\sigma}\left(t+\frac{r}{c}\right) - \tanh\frac{\pi}{\sigma}\left(t-\frac{r}{c}\right)\right] - \frac{q\pi}{2\sigma cr}\left[\operatorname{sech}^2\frac{\pi}{\sigma}\left(t+\frac{r}{c}\right) + \operatorname{sech}^2\frac{\pi}{\sigma}\left(t-\frac{r}{c}\right)\right] \tag{20}$$

$$N = -\frac{\partial\varphi}{c\partial t} = -\frac{q\pi}{2\sigma rc}\left[\operatorname{sech}^2\frac{\pi}{\sigma}\left(t+\frac{r}{c}\right) - \operatorname{sech}^2\frac{\pi}{\sigma}\left(t-\frac{r}{c}\right)\right]. \tag{21}$$

As t and $r \to 0$, $\varphi \to \frac{q\pi}{\sigma c}$ and $E_r \to 0$.

When r is great compared with σ, E_r and N have sharp maxima on the null-cone of magnitudes $-\frac{q\pi}{2\sigma cr}$ and $\pm\frac{q\pi}{2\sigma cr}$ respectively, N being negative on the incoming cone and positive on the outgoing. The momentum transfer in the vicinity of the null-cone is

$$\frac{1}{4\pi}\int_{-\infty}^{-0} EN4\pi r^2 dt$$

and for r sufficiently great is represented by $\frac{\pi}{3}\frac{q^2}{\sigma c^2}$. The energy stored in the field at time t is

$$\frac{1}{8\pi}\int_0^\infty (E^2 + N^2)4\pi r^2 dr = \frac{\pi}{3}\frac{q^2}{\sigma c}, \tag{22}$$

which is independent of the time. It should be noted that at $t = 0$, N is everywhere zero and the energy of the field derives completely from the electric force.

We now attempt to represent the inertial mass of the singularity. Imagine an electrostatic field of strength E parallel to x imposed at some $t < 0$ on the system we have just been studying. Because of the presence of the N-component in the field of the singularity, there will be momentum transfer parallel to x, per second, per unit area perpendicular to x of amount

$$\frac{E}{4\pi}\cdot\frac{q\pi}{2\sigma cr}\operatorname{sech}^2\frac{\pi}{\sigma}\left(t+\frac{r}{c}\right).$$

In a time interval sufficiently great compared with σ, the additional momentum parallel to x, $\frac{E}{4\pi}\cdot\frac{q}{rc}$, will have passed through the sphere $r = -ct$. In the

same time the rate of radial transfer of momentum (singularity alone) is

$$-\frac{1}{4\pi r^2}\frac{\pi}{3}\frac{q^2}{\sigma c^2} \text{ (per unit area).}$$

Consequently momentum is being transferred parallel to x with velocity

$$\frac{dx}{dt} = -c\frac{E}{4\pi}\frac{q}{rc}\Big/\frac{1}{4\pi r^2}\frac{1}{3}\frac{q^2\pi}{\sigma c^2} = -\frac{Eqr}{\frac{\pi}{3}\frac{q^2}{\sigma c^2}},$$

i.e.,

$$\frac{d^2x}{dt^2} = -\frac{Eq}{mc}\frac{dr}{dt} = \frac{Eq}{m}, \text{ with } m = \frac{\pi}{3}\frac{q^2}{\sigma c^3}. \tag{23}$$

Thus the singularity is accelerated like an electric point charge q of mass $m = \frac{\pi}{3}\frac{q^2}{\sigma c^3}$. This mass may be expected to determine the period of pulsation of energy mentioned above by means of de Broglie's relation $T = h/mc^2$.

General Considerations on the Nature of the Singularities in the Electromagnetic Field

On the basis of the foregoing study, one can obtain a certain new insight into the structure of electromagnetic field singularities.

(i) The model discussed exhibits the essential separation of space–time into two regions in one of which the field is static and in the other it vanishes. These regions are separated by the null-cone through the position of the singularity. The vicinity of the null-cone is the seat of very rapid change—quasi-discontinuity—in the electromagnetic potential and very high values of the electric field strength are attained on the null-cone itself.

(ii) The suddenness of the jump on the null-cone is determined by the time interval σ, which in turn also determines the mass of the singularity.

(iii) The vicinity of the null-cone is the seat of energy mc^2 and momentum mc.

(iv) The potential is everywhere finite as is also the electric force, and the latter is zero at the vertex of the null-cone.

(v) It should be noted, of course, that this type of field is made possible by introducing diagonal components into the electromagnetic field tensor.

We may say then, that the particle consists of a boundary of the field. The sharpness of the boundary is determined by (or determines) the mass of the particle. Except in the processes of particle creation or annihilation, this boundary is made up of pieces of null-cone of temporal duration $h/2mc^2$ or $hc/2\pi e^2$ times the effective thickness of space–time in which rapid changes of potential occur normal to the null-cone. The creation of a particle may take place from any surface in space, the field being established across the envelope of the null-cones expanding to infinity and the null-cones contracting to the position of the particle. Thus we may say that the particles are outside

of their electromagnetic fields in space–time. Any apparent departure from this rule is due to inherent indefiniteness of the boundary.

Connection with Quantum Theory

In the quantum theory it is necessary to apply electromagnetic representations with a certain vagueness. We use, for instance, the classical point-charge singularity, but we cannot connect it precisely with a space–time representation by giving the position and motion of the singularity in the classical way. We may regard the singularities in similar fashion; namely, we have an electromagnetic field representation of the singularity but it is to be applied via the uncertainty principle. Two types of possibility open up for us in this connection;

(*a*) With the wave-group representation of a particle in mind, we may imagine the successive appearances of an electron to be represented by the potential (19) centred on each of the discrete possibilities determined by the group of waves with the usual uncertainties of position and time prescribed by quantum theory; or

(*b*) We may imagine the uniform representation of the charge singularity as a pulsating boundary to the electromagnetic field to be applied as a substitute for the classical potential q/r in the quantum theory calculus of probability.

The representations with which we have dealt correspond to the static fields of classical theory. If we deal with a moving singularity, the resultant field is determined by Lorentz transformation, and it is easily seen that the consequences of the transformation are the same as in de Broglie's 'Ondes et Mouvements'. Lorentz transformation with velocity $u<c$ requires the field boundary in the neighbourhood of the singularity to move with velocity c^2/u. The model (*b*) above therefore can be connected with the de Broglie waves. We may think of the boundary of the field as propagated like electromagnetic waves in a wave-guide. The pulsations of energy can be regarded as analogous to the transverse pulsations of energy in the guide. Only in our model, not only does the magnetic field enter to determine energy and momentum flow in the field, we also require the field component N which acts as a substitute for the currents which would be required in Maxwell–Heaviside theory to support the waves.

Not only should we, according to the present view, think of an electron as a field boundary: a light quantum and a neutrino should be thought of similarly. To support it the field boundary requires the new field component N by which energy and momentum can be radiated non-optically, and which, in acting on a particle, can induce change in its rest-mass i.e., this field component is associated with the creation and annihilation of field singularities.

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